Velocity fluctuations in a turbulent soap film: The third moment in two dimensions

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Quasi-two-dimensional decaying turbulence is studied in a flowing soap film by measuring the moments of the probability density function $P(\delta v(r))$ for the longitudinal velocity differences $\delta v(r)$ on a scale r. As in three-dimensional (3-D) turbulence, P becomes non-Gaussian with decreasing r. The third moment $S_3(r)\equiv\langle(\delta v(r))^3\rangle$ is small and negative at small scales, but becomes positive at larger scales. The exact calculation of $S_3(r)$ for 2-D homogeneous isotropic turbulence relates this change in sign to the development of the velocity correlation function as the turbulence decays. [S1070-6631(99)01905-4]

I. INTRODUCTION

It is generally believed that the Navier–Stokes equation governs turbulence in Newtonian fluids. However, only a few rigorous results relevant to incompressible isotropic and homogeneous turbulence can be derived directly from this equation. The most famous one pertains to the third moment of the difference in velocity between two points. For the component of this velocity difference $\delta\mathbf{v}(\mathbf{r})$ projected along their separation $\mathbf{r}$ [the longitudinal velocity differences $\delta v(r)$], Kolmogorov showed that in three dimensions.[1–3]

$$S_3(r) = -\frac{4}{5}\varepsilon r + 6\nu\frac{dS_2(r)}{dr}, \tag{1}$$

where $S_n(r)\equiv\langle(\delta v(r))^n\rangle$, ε is the rate of energy flux through the system, and ν is the kinematic viscosity; the brackets here denote an ensemble or time average. In the range where viscosity is unimportant, this equation gives Kolmogorov's $\frac{4}{5}$ law: $S_3(r) = -\frac{4}{5}\varepsilon r$.[1] This relation seems fairly well confirmed by experiment.[2–4]

In this paper we report on measurements of the second and third moments of $\delta v(r)$ in a rapidly flowing and freely suspended turbulent soap film. This system exhibits some aspects of decaying two-dimensional (2-D) turbulence;[5–7] the film is only a few microns thick, which is much smaller than its lateral dimensions. Our results indicate that, although the same equations which apply to 3-D turbulence can be used in modified form for two dimensions, the balance of the terms is not the same: the difference between forced and decaying turbulence is apparently much more important in two dimensions than in three dimensions. The most striking effect is that $S_3(r)$ becomes positive at sufficiently large r before eventually decaying to zero, as indeed it must. Our measurements show that this change in sign of S_3 is related to the crossing of the velocity correlation functions as the turbulence decays downstream. This may well be related to the transfer of energy to larger scales. In 3-D decaying turbulence, $S_3(r)$ is observed to be negative at all r.[4,8]

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II. BACKGROUND AND SETUP

Soap films have long been known as a source of fascinating and beautiful phenomena,[9] but it is only recently that Couder *et al.*[10,11] and Gharib and Derango[5] demonstrated that the hydrodynamics of these films are strikingly two-dimensional. Quasi-2-D turbulence can also occur due to the effects of rotation, density stratification, or electromagnetic forces.[4,12,13] Thus turbulence in two dimensions is not only the province of the computer.

The fundamental difference between 2-D and 3-D turbulence is that in three dimensions vorticity can be amplified by velocity fluctuations, whereas in two dimensions vorticity can only dissipate.[4] Thus the vorticity $\omega(\equiv\nabla\times\mathbf{v})$, or equivalently the enstrophy $\Omega(\equiv\frac{1}{2}\langle\omega^2\rangle)$ becomes a nearly conserved quantity in two dimensions, like the energy. In the cascade picture for 3-D turbulence,[2,3,14] which applies in the inertial range where viscosity is unimportant, kinetic energy is transferred from larger to smaller scales. In this range, velocity fluctuations $\delta v(r)$ on a scale r depend only on ε and r itself; dimensionally, $\delta v(r)\sim(\varepsilon r)^{1/3}$, which is consistent with but not implied by Eq. (1). In two dimensions it is expected that there is a *direct* cascade of enstrophy to smaller scales, and an *inverse* cascade of energy to larger scales;[15] for decaying 2-D turbulence the inverse cascade is appar-

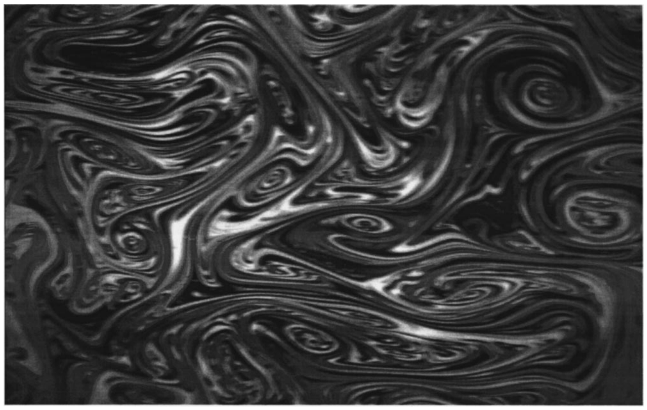

FIG. 1. The central region of the turbulent soap film, $Y=8$ cm behind comb. The image is about 4 cm across, and the flow is from right to left. The mean flow speed is 180 cm/sec.

ently absent.[16] The enstrophy transfer rate β (Ω per unit time) in the direct cascade leads dimensionally to $\delta v(r) \sim (\beta)^{1/3} r$. Although this picture seems to be confirmed by numerical simulations[4,17–20] (but see Ref. 21), the experimental situation is still evolving.[22–24]

The measurements reported here were made in a 2.4 m high soap film apparatus developed in our laboratory.[6,25] A solution of soap and water (2% commercial liquid detergent by volume) is introduced at a constant rate between two nylon wires and allowed to fall under its own weight as a film approximately 3 μm in thickness. The width of the channel is $W=6.2$ cm over a distance of 120 cm. Nearly isotropic turbulence[7] is produced in this region by a comb inserted perpendicularly into the film, having a 1 mm tooth diameter and a spacing $M=3.8$ mm. An interferometric image of the turbulence thus produced is shown in Fig. 1.

To measure the time variation of the film velocity, we use laser Doppler velocimetry.[26] The commercial apparatus (TSI Inc.) produces a patch of fringes roughly 35 μm in size. The flow is seeded with 1 μm polystyrene spheres, and the average data rate is about 8 kHz. At a distance $Y \simeq 8$ cm below the comb, where most of our measurements were made, the mean and rms velocities were typically $U_0 = 180$ cm/sec and $v_{\rm rms} \equiv \langle v'^2 \rangle^{1/2} = 24$ cm/sec, where $v' \equiv v - U_0$. The actual value of the viscosity for soap film flows is not completely clear; in this paper we use $\nu = 0.1$ cm^2/sec.[27] The Reynolds number of the channel is $\mathrm{Re}_W = U_0 W/\nu \simeq 11\,000$, and for the comb $\mathrm{Re}_M \equiv U_0 M/\nu \simeq 700$. These numbers are comparable to those obtained in experiments on 3-D grid turbulence,[8,28] and thus we consider our flow to be turbulent in some generic sense. The deviations from two-dimensionality due to air friction[25] appear not to affect the turbulence for the scales of interest here.[7]

III. RESULTS

We measure the longitudinal velocity difference $\delta v(r,t) \equiv (\mathbf{v}(\mathbf{x}+\mathbf{r},t) - \mathbf{v}(x,t)) \cdot \mathbf{r}/r$, with $\mathbf{r}$ in the direction of the flow y. Because $v_{\rm rms}/U_0$ is only about 0.14, we are justified in using Taylor's hypothesis, and accordingly use $y = U_0 t$ to relate time correlation functions to spatial ones.[3] It has been verified that Taylor's hypothesis holds in our soap film for moments out to $S_6(r)$;[29] we have also taken into account the weak variation of U_0 and $v_{\rm rms}$ with Y in applying this hypothesis.

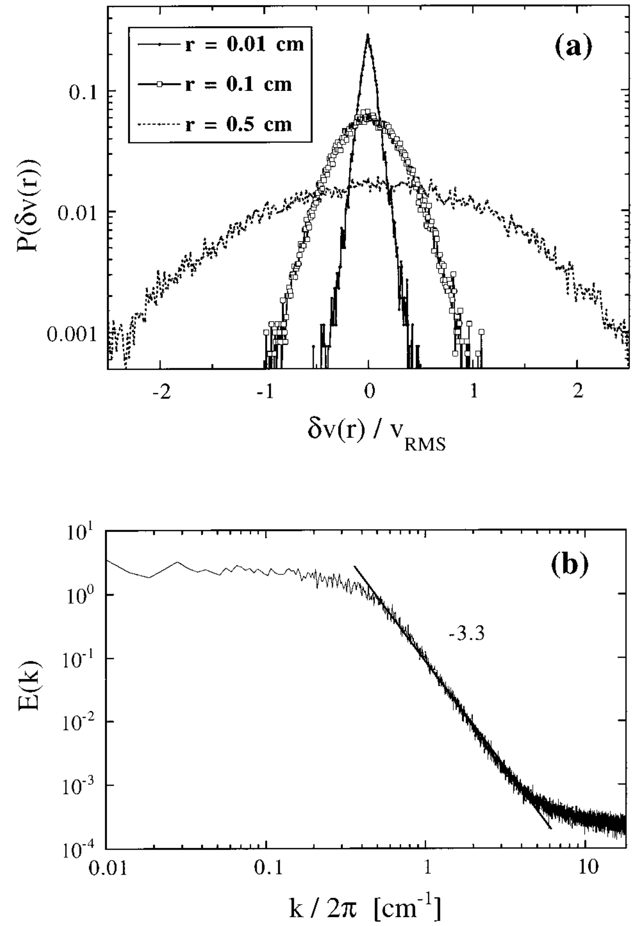

FIG. 2. (a) The PDFs of $\delta v(r)$ at the three indicated values of r. (b) Energy spectrum obtained from the time series of the longitudinal velocity, showing an interval where $E(k) \sim k^{-3.3}$. The data in both plots are taken at $Y = 8$ cm.

Figure 2(a) is a semi-log plot of the normalized probability density functions (PDFs) of δv for several different spatial separations r, at $Y=8$ cm. Note that as r decreases, $P(\delta v(r))$ changes shape from Gaussian at large scales ($r>2$ mm) to double-sided exponential at small scales ($r<0.1$ mm). The development of extended tails at small r indicates the intermittency[3] of the fluctuations, similar to 3-D turbulence.[30,31]

Figure 2(b) shows the power spectrum $E(k)$ for the same data. Over a little less than a decade $E(k) \sim k^{-3.3}$, in agreement with previous measurements;[5–7,23] this exponent does not change measurably with Y. This observation is consistent with the expected scaling for an enstrophy cascade, $E(k) \sim k^{-3}$.[15,16] The absence of a $k^{-5/3}$ portion at small k is consistent with the current understanding of decaying 2-D turbulence.[16,32]

It should be noted here that a k^{-3} scaling has been observed in numerical simulations of forced 2-D turbulence in the inverse energy cascade range as well.[18,21] This deviation from the expected $k^{-5/3}$ scaling is attributed to the presence of large, long-lived coherent structures and finite-size effects

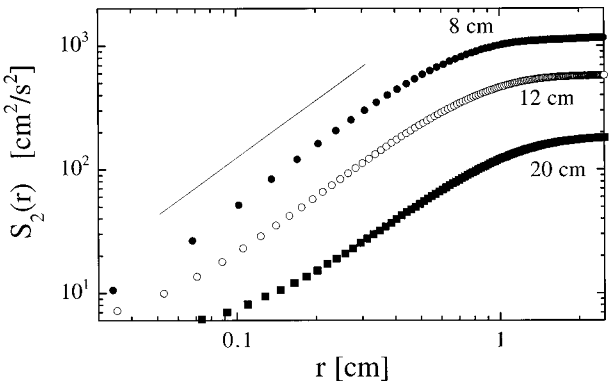

FIG. 3. Log–log plot of $S_2(r)$ vs. r at the indicated distances (Y) below the comb; the solid line corresponds to $S_2(r) \propto r^{1.6}$.

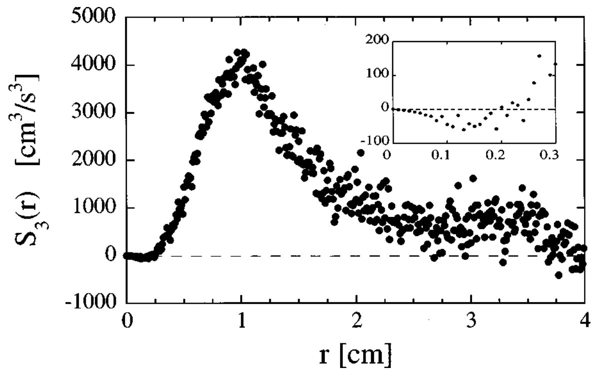

FIG. 4. The third moment $S_3(r)$ (linear scale) measured at $Y = 8.0$ cm; the dashed line is $S_3(r) = 0$. The inset is an enlargement at small r.

in the flow.[18] Thus our observation of a k^{-3} scaling may not uniquely characterize an enstrophy cascade (though see Ref. 7).

From the distributions $P(\delta v(r))$ one obtains the moments. Figure 3 shows the second moment $S_2(r)$ at several different Y. In the enstrophy cascade region one expects on dimensional grounds that $S_2(r) \sim \beta^{2/3} r^2$.[4] Although we observe a scaling region of about one decade in r, we find $S_2(r) \sim r^{1.6 \pm 0.2}$. However, this is not necessarily a disagreement. As pointed out by Babiano *et al.*,[33] the limited *width* of a scaling range in $E(k)$ can reduce the scaling exponent by effectively introducing other length scales: the bounds of the scaling region. There is also no reason why these bounds should be the same in $E(k)$ or $S_2(r)$.[34] Moreover, it is not clear which function should give the more physically relevant bounds for this scaling range. Because our measurements are made in real space (via Taylor's hypothesis), we will define the scaling range based on $S_2(r)$. In this paper we focus on the position $Y = 8$ cm, for which the scaling range is roughly from 0.04 cm to 0.3 cm.

The second moment $S_2(r)$ is often used to define an integral scale:[2,4] $\ell_0 \equiv \int_0^\infty b(r)\, dr / v_{\mathrm{rms}}^2$, where

$$b(r) \equiv \langle v'(x) v'(x+r) \rangle = v_{\mathrm{rms}}^2 - \tfrac{1}{2} S_2(r) \tag{2}$$

is the velocity correlation function. For the $Y = 8$ cm data in Figs. 2(a) and 2(b), we find $\ell_0 = 0.6$ cm, and the Reynolds number $\mathrm{Re}_\ell \simeq 1000$. Note that at $r \simeq 2\ell_0$, $S_2(r)$ has flattened out and the velocity fluctuations are uncorrelated ($b \simeq 0$). A discussion of 2-D turbulence at such Reynolds numbers is given in Ref. 32.

We focus our attention on the third moment $S_3(r) = \langle (\delta v)^3 \rangle$ shown in Fig. 4. In these measurements, made at $Y = 8$ cm, we see immediately that the dependence of $S_3(r)$ is completely different than in three dimensions, where it is negative for all r.[4] In the range $r < 0.2$ cm, we find that $S_3(r)$ is negative but very close to zero (see inset to Fig. 4); this is approximately the same as the scaling range for $S_2(r)$ (see Fig. 3). At approximately 0.2 cm, $S_3(r)$ crosses zero and becomes positive, increasing with r up to about 1 cm, where it reaches a maximum. As r increases further, velocity fluctuations become uncorrelated, so that $P(\delta v(r))$ becomes Gaussian and $S_3(r) \to 0$ in the limit of large r. All of this occurs in a range where $S_2(r)$ has at most a very weak dependence on r. The crossover from a small negative $S_3(r)$ to positive values at larger r is a robust result, reproduced at various flow speeds and several different experimental setups. It is interesting to note that a recent numerical simulation of 2-D turbulence has shown that $S_3(r)$ can be positive as well as negative.[35]

The qualitative shape of $S_3(r)$ shown in Fig. 4 is also seen further downstream. However, the crossing point and the maximum in S_3 increase with downstream distance Y, as does the integral scale ℓ_0. To understand this, consider the maximum value of $S_3(r)$ occuring at $r = r_{\max}$ (about 1 cm); note that $r_{\max} \simeq \ell_0$. As the turbulence decays below the comb, time is required to transfer energy from the scale M on which it is generated to larger scales. Therefore, at any Y there are no vortices larger than some size $\sim r_{\max}$, which should increase with Y. Denoting the transit time $\tau = Y/U_0$, the data indeed show that $r_{\max}$ increases with τ. Though we cannot say with precision what the actual dependence is, the increase appears to be slower than linear, a form predicted for decaying 2-D turbulence.[16]

Can we understand the $S_3(r)$ that we have measured in our turbulent soap film in terms of the 2-D Navier–Stokes equation? The derivation of Eq. (1) follows from the von Karman–Howarth equation,[2,3,8] and is exact for continuously forced turbulence in three dimensions; it is only in this case that ε is a unique quantity, whether it is considered as the energy injection rate, spectral transfer rate, or dissipation rate. For decaying 3-D turbulence, Eq. (1) is generally believed to hold, and experimentally the turbulence seems to be tolerably close to homogeneous forced turbulence.[28] However, in two dimensions there is no theoretical expectation for $S_3(r)$,[36–38] though it has recently been considered for 2-D magnetohydrodynamic turbulence.[39]

To explain our surprising observations, we have rederived the exact relation between $S_3(r)$ and $S_2(r)$ for decaying but locally stationary 2-D turbulence by modifying the corresponding equation for three dimensions [Eq. (33.17) in Ref. 2)]. We write this equation in the following form:

$$\frac{\partial}{\partial r}(r^3 S_3) = 6 r^3 \frac{\partial b}{\partial \tau} + 6\nu \frac{\partial}{\partial r}\left(r^3 \frac{\partial S_2}{\partial r} \right), \tag{3}$$

where $\partial b / \partial \tau = U_0 (\partial b / \partial Y)$ is the downstream decay of $b(r)$.

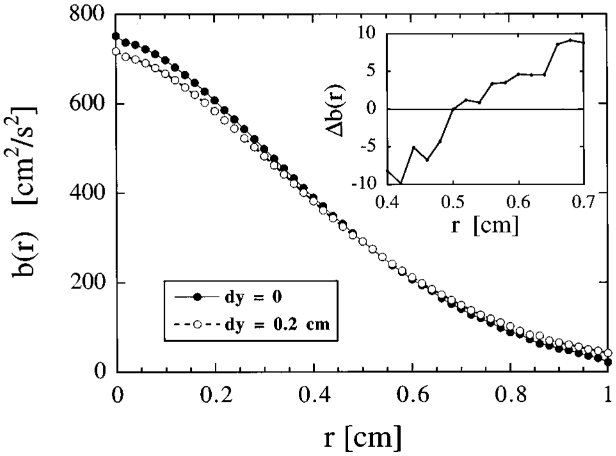

FIG. 5. The velocity correlation function $b(r)$ measured at two different downstream distances, separated by 0.20 cm, at $Y=8.6$ cm. The inset shows the difference Δb of these two curves near their crossing point; note that $\Delta b>0$ at large r.

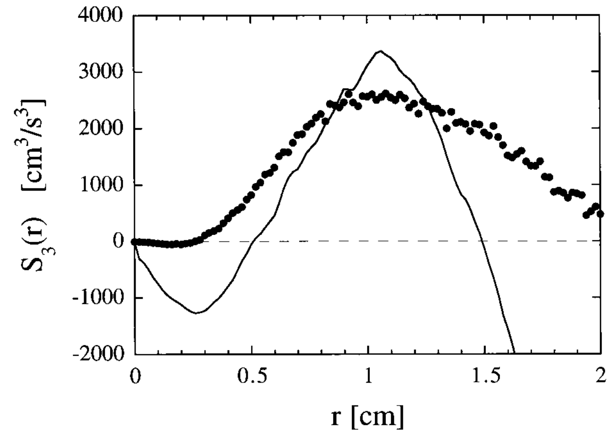

FIG. 6. The calculated $S_3(r)$ using the data from Fig. 5 in Eq. (5); the solid circles correspond to $S_3(r)$ measured at the same position, $Y=8.6$ cm.

From the definition of $b(r)$, and the fact that the total kinetic energy per unit mass in two dimensions is $v_{\rm rms}^2$,[40] it follows that

$$U_0 \frac{\partial b(r,Y)}{\partial Y} = -\varepsilon(Y) - \frac{U_0}{2}\frac{\partial S_2(r,Y)}{\partial Y}, \tag{4}$$

where

$$\varepsilon(Y) = -\frac{1}{2}\frac{\partial}{\partial \tau}\langle v_x^2 + v_y^2\rangle = -\frac{\partial\langle v_y^2\rangle}{\partial \tau} = -U_0 \frac{\partial v_{\rm rms}^2(Y)}{\partial Y}$$

for the isotropic case.

For 3-D decaying turbulence, one usually neglects the term $\partial S_2/\partial Y$, and obtains Eq. (1).[2] In the forced case $\partial S_2/\partial\tau=0$, but a different derivation of Eq. (1) is needed.[3] For 2-D decaying turbulence a derivation following the 3-D case leads to $S_3(r) = -\frac{3}{2}\varepsilon r$. However, in our experiment we find that the decrease of $S_2(r)$ with downstream distance Y is not negligible (see Fig. 3), so $\varepsilon(Y)$ no longer dominates the right-hand side of Eq. (4). We therefore prefer to integrate Eq. (3) directly and obtain the expression

$$S_3(r) = \frac{6}{r^3}\int_0^r U_0 \frac{\partial b(x,Y)}{\partial Y} x^3\,dx + 6\nu \frac{dS_2(r)}{dr}. \tag{5}$$

By measuring $b(r)$ at two closely spaced values of Y in the neighborhood of $Y=8$ cm, we approximate $\partial b/\partial Y \simeq \Delta b/\Delta Y$ and numerically obtain the integral in Eq. (5). The two $b(r)$ are shown in Fig. 5, for $\Delta Y = 0.20$ cm. At $r=0$ this implies $\varepsilon = 1.3\times 10^4\ {\rm cm^2/sec^3}$, which agrees with the estimate $\varepsilon \simeq v_{\rm rms}^3/\ell_0 = 2\times 10^4\ {\rm cm^2/sec^3}$.[3] The crossing of the correlation functions (see inset to Fig. 5) at $r\simeq 0.5$ cm is responsible for the change in sign of $S_3(r)$, as indicated by Eq. (5). This is consistent with the fact that in 3-D decaying turbulence the $b(r)$ are not seen to cross.[28,41,42] The calculated $S_3(r)$ using Eq. (5) is shown in Fig. 6, without the viscous term, which was estimated to have a small effect. Also shown is the measured $S_3(r)$.

IV. DISCUSSION AND CONCLUSION

The semi-quantitative agreement of the calculated and measured $S_3(r)$ indicates that Eq. (5) may indeed be valid for 2-D decaying turbulence. The deviation of this equation from the data could be due to several factors. First, it is possible that we have not entirely captured the derivative $\partial b/\partial Y$ by our approximation $\Delta b/\Delta Y$. Alternatively, the turbulence may not be homogeneous enough or sufficiently isotropic at large scales, though it is isotropic at small scales.[6,7] In addition, the flow may simply not be ''turbulent enough,'' in the sense of having a large enough Re to have well-developed inviscid transfer ranges. It is also possible that the soap film, though very thin, may not completely mimic a 2-D incompressible system.[24] This compressibility is due to the moderate speed of capillary waves in the film,[11] which results in the production of fluctuations in thickness. These elastic effects can be included in a Navier–Stokes equation for soap film flow,[11] but it is not evident how that would affect the above derivation.

An intriguing interpretation of our measured third moment is suggested by the relationship between $S_3(r)$ and a locally defined energy transfer rate $\varepsilon(r)$.[43] The association of the energy transfer variable with a particular *spatial* scale r is fundamentally different from our use of ε up to this point. But because this scale-dependent energy transfer rate is completely determined by $S_3(r)$, it permits us to compare our experimental measurements to standard results from turbulence models. If we interpret the scaling range of $S_2(r)$ $(0.04\ {\rm cm} < r < 0.30\ {\rm cm})$ as an enstrophy cascade range, then our observation that $S_3(r)$ is approximately zero in this range would be consistent with the absence of energy transfer for these scales $[\varepsilon(r)\simeq 0]$.[15] Furthermore, the region where $S_3(r)>0$, corresponding to $\varepsilon(r)<0$, would then indicate the *inverse transfer* of energy to larger scales. This interpretation implies a prediction for fully developed forced 2-D turbulence: $S_3(r)$ should be linear and positive in the inverse cascade range. These ideas remain to be checked in future experiments and in a more rigorous theoretical treatment of the third moment for 2-D turbulence.

We cannot rule out the possibility that the scaling range observed here, in which $S_2(r) \sim r^{1.6}$, may be due to a complex inverse cascade regime produced by the accumulation of energy at large scales, as has been observed in some numerical simulations.[18,21] However, there is other experimental evidence which suggests that this range does indeed correspond to the expected enstrophy cascade.[7,24] The question remains open.

Our study has concentrated experimentally on the r dependence of the third moment $S_3(r)$ in a turbulent soap film. Despite the large number of simulations of 2-D turbulence, there are almost no studies with which we can compare our observations. In two dimensions as well as in three dimensions, an exact equation relates the second and third moments. The particular nature of 3-D turbulence permits the application of the theory of forced homogeneous isotropic turbulence to the turbulence generated by a grid in wind or water tunnels, even though this turbulence is decaying. Our experimental results suggest that the situation in two dimensions is very different.

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